ID: 725 Problem ①

From given, we have $\frac{1}{a} + \frac{a}{b} + \frac{1}{ab} = 1$. And since $a, b \in \mathbb{Z}^+$, then $a, b, ab \neq 0$

Multiply both sides by ab, we get

$$b + a^2 + 1 = ab$$

$$a^2 - ab + b + 1 = 0$$

$$a^2 - ab + b - 1 = -2$$

$$(a-1)(a-b+1) = -2$$

Since $a, b \in \mathbb{Z}^+$, then $a-1$ and $a-b+1$ must be integer factor of -2.

The possibilities are $(-2)\cdot 1$, $1\cdot(-2)$, $2\cdot(-1)$, $(-1)\cdot 2$.

Case 1: $\begin{cases} a-1 = -2 \\ a-b+1 = 1 \end{cases} \Rightarrow \begin{array}{l} -b+2 = 3 \\ b = -1 \ngtr 0 \end{array}$ So case 1 does not work

case 2: $\begin{cases} a-1 = 1 \\ a-b+1 = -2 \end{cases} \Rightarrow \begin{array}{l} -b+2 = -3 \\ b = 5 > 0 \end{array} \Rightarrow \begin{array}{l} a-5+1 = -2 \\ a = 2 > 0 \end{array}$ ✓ $(2, 5)$ works

case 3: $\begin{cases} a-1 = 2 \\ a-b+1 = -1 \end{cases} \Rightarrow \begin{array}{l} -b+2 = -3 \\ b = 5 > 0 \end{array} \Rightarrow \begin{array}{l} a-5+1 = -1 \\ a = 3 > 0 \end{array}$ ✓ $(3,5)$ works

case 4: $\begin{cases} a-1 = -1 \\ a-b+1 = 2 \end{cases} \Rightarrow \begin{array}{l} -b+2 = 3 \\ b = -1 \ngtr 0 \end{array}$ so case 4 does not work

There fore, $\boxed{(2,5), (3,5)}$ are the only ordered pair of positive integer (a,b).